MTH 344 Midterm Review, Fall 2009

1. For each pair G, S below, decide whether or not the set on the right is a subgroup of the group on the left. Justify your answer.
   The set of real numbers is denoted by **R** and **R*** denotes the set of nonzero real numbers.

   a) G = **R*** with multiplication — S = {-1,0,1}

   b) G = {f: **R**→**R** : f is 1-1 and onto} with function composition — S = {f∈G : f(1) = 2}

   c) G = Rot_{12} with function composition — S = {I, R^4, R^8} where R is a 30° clockwise rotation

2. Let a be an element of a group G. Prove: $(a^{-1})^{-1} = a$

3. Give an example of each of the following:

   a) A group G where G has 1000 elements

   b) An infinite group with a nontrivial finite subgroup

   c) A non-commutative group with a commutative subgroup

4. Let G be a commutative group and let a,b∈G such that o(a) = 2 and o(b) = 3.

   a) Prove: o(ab) = 6

   b) Give a counterexample showing that if G is not commutative, then part a) might not hold.

5. Let G be a commutative group with subgroups H and K.
   Let S = {hk : h∈H and k∈K}

   a) Prove that S is a subgroup of G.

   b) Give a counterexample showing that if G is not commutative, then part a) might not hold.

6. Let G be a group with identity element e and let a,b,c be elements of G.
   Prove: If abc = e, then cab = e