

Math 344 Fall 09
Homework #3
Due Thursday, October 22, 2009

1. **Two Important Function Properties:** Some functions are *1-1* and/or *onto*. (*Bijjective* functions are both.) Here are definitions for these terms.

Let $f: S \rightarrow T$ be a function.

1-1: No element of T is hit by two different elements of S .

Formally, $f(s_1) = f(s_2)$ implies $s_1 = s_2$ for all $s_1, s_2 \in S$.

onto: Each element of T is hit by some element of S .

Formally, for each $t \in T$, there is an element $s \in S$ such that $f(s) = t$.

A) Suppose $g: A \rightarrow B$ and $f: B \rightarrow C$ are functions.

Prove that if f and g are both 1-1, then $f \circ g$ is 1-1.

B) Suppose $g: A \rightarrow B$ and $f: B \rightarrow C$ are functions.

Prove that if f and g are both onto, then $f \circ g$ is onto.

C) Give an example to show that $f \circ g$ 1-1 does not imply that f and g are 1-1.

D) Give an example to show that $f \circ g$ onto does not imply that f and g are onto.

Hint: For C) and D), use very small sets (no more than 3 elements) to give your examples and use arrow diagrams to define your functions, not formulas.

2. Let G be a group with the following property: If $a, b,$ and c belong to G and $ab=ca$, then $b=c$. Prove that G is abelian.

3. Definition: Let G be a group and H a subset of G . H is a *subgroup* of G if it is a group with respect to the operation defined on G .

a. List all of the subgroups of D_6 (the group of symmetries of an equilateral triangle.)

b. List all of the subgroups of D_8 (the group of symmetries of a square.)

c. List all of the subgroups of $(\mathbb{Z}, +)$.
