Math 344 Fall 09
Homework #5
Due Tuesday, November 24, 2009

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1. Prove both of the following theorems about isomorphisms.

Theorem: Suppose ***G*** and ***H*** are isomorphic groups then, if ***G*** is cyclic, then ***H*** is cyclic. (Recall a group is cyclic if there exists an element **a** such that for every element **g** in ***G***, there exists some integer **i** such that $\mathbf{g}=\mathbf{a}^{\mathbf{i}}$.)

Theorem: Suppose (***G***, •) and (***H***, ∗) are groups and φ: ***G***→***H*** is an isomorphism. Let ***a*** ∈ ***G***. Then, $|\varphi(a)| = |a|$. (Recall the order $|a|$ of an element ***a*** is the least positive integer ***i*** such that a^i is the identity).

2. Use induction to prove that for any nonnegative integer n,

$$1 + 2 + 2^2 + 2^3 + ... + 2^n = 2^{n+1} - 1.$$
