

# Math 344 - Midterm Exam

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YOUR NAME.....

problem	score
1	
2	
3	
4	
5	
6	
7	
8	
total	

**Work each of the 8 problems.**

**Show all of your work.**

**Note:** Your answers will be graded for correctness and clarity, so show all of your steps and explain your reasoning. Some tips:

- (1). Work neatly and circle your answers.
- (2). State any assumptions you are making.
- (3). Explain your answers... (more important than simplifying).

1. Define *group*.

2. Define the *order* of a group element.

3. Define  $\circ$  on the set  $\mathbb{R}^+$  of positive real numbers by

$$x \circ y = x^2 y.$$

Is  $(\mathbb{R}^+, \circ)$  a group?

4. Let  $G = \{x \in \mathbb{R} \mid x \neq -1\}$ , and define  $a * b$  so that

$$a * b = a + b + ab.$$

Determine whether  $(G, *)$  is a group.

5. Let  $G$  be a group and let  $a, b \in G$ . Define a function  $f : G \rightarrow G$  by

$$f(x) = axb.$$

Determine whether  $f$  is 1-1 (injective) and/or onto (surjective).

6. Let  $G$  be an **abelian** group and assume  $H$  is a subgroup of  $G$ . Let

$$K = \{x \in G : x^2 \in H\}.$$

Must  $K$  also be a subgroup of  $G$ ?

7. Prove or disprove: If  $g^2 = e$  for all  $g \in G$ , then  $G$  is abelian.

8. Prove or disprove: If  $a^2b^2 = b^2a^2$  for all  $a, b \in G$ , then  $G$  is abelian.