

Math 344 - Day 8: More about subgroups

After discussing questions about homework, we return to the topic of subgroups. Since subgroups can tell us a lot about the structure of a group, we spend a day exploring some important types of subgroups.

Intersections and Unions. Explore whether and when the union and/or intersection of two or more subgroups is again a subgroup.

Cyclic Subgroups. Let G be a group with identity e . If $a \in G$, then the **cyclic subgroup generated by a** is defined to be:

$$\langle a \rangle := \{a^i \mid i \in \mathbb{Z}\}$$

where $a^0 := e$ and where negative powers of a are interpreted as being the corresponding powers of a^{-1} . What is the order of $\langle a \rangle$?

The Center of a Group. Let G be any group. The **center** of G is the set

$$\mathbf{Z}(G) := \{a \in G \mid \forall b \in G \ ab = ba\}.$$

Prove that the center is always a subgroup of G . Find the centers of some familiar groups.
