

Math 344 - Day 9: More about subgroups

We begin with a content survey from the curriculum designers, which we follow with a brief review. Then, after discussing questions about homework, we return to the topic of subgroups. Since subgroups can tell us a lot about the structure of a group, we spend (another) day exploring some important types of subgroups.

Cosets of a Subgroup. Suppose H is a subgroup of G . Fix any element $a \in G$ and define the set

$$aH := \{ah \mid h \in H\}.$$

Such a set is called a (left) **coset of H** . Prove that, in general, such sets are NOT subgroups of G . Can they ever be? Find some cosets of some familiar subgroups.

Points to consider. How many elements are in a coset of a subgroup? Is the collection of right cosets of a subgroup identical to the collection of left cosets of that subgroup? If a and b are different elements, can they give the same coset?

Conjugation of a Subgroup. Suppose H is a subgroup of G . Fix any element $a \in G$ and define the set

$$a^{-1}Ha := \{a^{-1}ha \mid h \in H\}.$$

Such a set is called a **conjugate H** . Prove that any conjugate of H will be a subgroup of G .
