

## Math 344 - Day 9: More about subgroups

We begin with a content survey from the curriculum designers, which we follow with a brief review. Then, after discussing questions about homework, we return to the topic of subgroups. Since subgroups can tell us a lot about the structure of a group, we spend (another) day exploring some important types of subgroups.

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**Cosets of a Subgroup.** Suppose  $H$  is a subgroup of  $G$ . Fix any element  $a \in G$  and define the set

$$aH := \{ah \mid h \in H\}.$$

Such a set is called a (left) **coset of  $H$** . Prove that, in general, such sets are NOT subgroups of  $G$ . Can they ever be? Find some cosets of some familiar subgroups.

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**Points to consider.** How many elements are in a coset of a subgroup? Is the collection of right cosets of a subgroup identical to the collection of left cosets of that subgroup? If  $a$  and  $b$  are different elements, can they give the same coset?

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**Conjugation of a Subgroup.** Suppose  $H$  is a subgroup of  $G$ . Fix any element  $a \in G$  and define the set

$$a^{-1}Ha := \{a^{-1}ha \mid h \in H\}.$$

Such a set is called a **conjugate  $H$** . Prove that any conjugate of  $H$  will be a subgroup of  $G$ .

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