Homework # 3: due 11/06/09

Consider the boundary value problem:

$$u''(x) = u'(x) + 2u(x) + \cos x, \quad 0 < x < \pi$$
$$u(0) = -0.3, \quad u(\pi) = 0.3$$

The exact solution is $u(x) = -(\sin x + 3\cos x)/10$.

Implement Jacobi's iterative method to solve this problem using a central finite difference discretization of the first and second order derivatives

$$\frac{u_{i-1} - 2u_i + u_{i+1}}{h^2} = \frac{u_{i+1} - u_{i-1}}{2h} + 2u_i + \cos x_i, \quad i = 1, 2, \ldots n-1$$

where

$$h = \pi/n, \quad x_i = i * h, \quad i = 0, 1, \ldots, n$$

PROVIDE:

(20 points) Serial (5 points) and parallel (15 points) code implementation. Provide the number of iterations required for $n = 1000\,(h = \pi/n)$, a tolerance $\epsilon = 10^{-6}$, and an initial guess from linear interpolation of the boundary values.

(5 points) A graph of the parallel code performance: for fixed $\epsilon = 10^{-3}$ and $n = 10000$ graph the *cpu* time for $np = 1, 2, 4, 8$.