

Homework #7, due 06/02, in class

Each problem is worth 5 points.

Problem 1. Find the solution of the linear system:

$$\begin{aligned}\frac{\partial u_1}{\partial t} + \frac{\partial u_2}{\partial x} &= 0 \\ \frac{\partial u_2}{\partial t} + 4\frac{\partial u_1}{\partial x} &= 0\end{aligned}$$

that satisfies the initial condition

$$\begin{aligned}u_1(x, 0) &= 2x \\ u_2(x, 0) &= -x\end{aligned}$$

Problem 2. Find the solution of the linear system:

$$\begin{aligned}\frac{\partial u_1}{\partial t} + 2\frac{\partial u_2}{\partial x} &= 0 \\ \frac{\partial u_2}{\partial t} + \frac{\partial u_1}{\partial x} + \frac{\partial u_2}{\partial x} &= 0\end{aligned}$$

that satisfies the initial condition

$$\begin{aligned}u_1(x, 0) &= x \\ u_2(x, 0) &= \sin(x)\end{aligned}$$

Problem 3. Find the general solution of the linear system

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}_t + \begin{bmatrix} 1 & 2 \\ 4 & -1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}_x = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Problem 4. Find the solution of the linear system

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}_t + \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}_x = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

that satisfies the initial condition

$$u_1(x, 0) = e^x, \quad u_2(x, 0) = x$$

Problem 5. Find the solution of the linear system

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}_t + \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}_x = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

that satisfies the initial condition

$$u_1(x, 0) = \phi_1(x), \quad u_2(x, 0) = \phi_2(x)$$

Problem 6. Find the solution of the linear system

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}_t + \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}_x = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

that satisfies the initial condition

$$u_1(x, 0) = \sin(x), \quad u_2(x, 0) = x$$