

Homework # 1, due 10/19, in class¹

To receive full credit, present complete answers that show all work.

Problem 1. Solve the initial value problem and determine the values of x and y for which the solution exists

(a-5points) $xu_x + u_y = y, \quad u(x, 0) = x^2$

(b-5points) $u_x + u^2u_y = 1, \quad u(x, 0) = 1$

Problem 2 (5points). Consider the Cauchy problem

$$y^2u_x + xu_y = \sin(u^2), \quad u(x, 0) = x$$

Determine the values of u_{xx}, u_{xy}, u_{yy} on the x -axis.

Problem 3 (10 points). Solve the initial value problem

$$a(u)u_x + u_y = 0, \quad u(x, 0) = h(x)$$

Show that the solution becomes singular for some $y > 0$ unless $a(h(s))$ is a nondecreasing function of s .

Problem 4 (5 points). Solve the initial value problem

$$u_x^2 + yu_y - u = 0, \quad u(x, 1) = \frac{x^2}{4} + 1$$

Problem 5 (5 points). Solve the initial value problem

$$u = xu_x + yu_y + \frac{1}{2}(u_x^2 + u_y^2), \quad u(x, 0) = \frac{1 - x^2}{2}$$

Problem 6 (10 points). Consider the Cauchy problem

$$u = u_x^2 + u_y^2, \quad u(x, 0) = ax^2$$

For what positive constants a is there a solution? Is it unique? Find all solutions.

¹All problems are from McOwen's book