Homework # 2, due 11/02, in class[1]

To receive full credit, present complete answers that show all work.

Problem 1 (5 points). (Section 2.1) Find the Taylor series solution about $(x, y) = (0, 0)$ of the initial value problem

$$u_y = \sin(u_x), \quad u(x, 0) = \frac{\pi x}{4}$$

Problem 2 (10 points). (Section 2.2) Find the general solution to the problem

$$u_{xx} - 2u_{xy} \sin x - u_{yy} \cos^2 x - u_y \cos x = 0$$

Problem 3 (10 points). (Section 2.2) Find the general solution to the problem

$$y^2 u_{xx} - 2y u_{xy} + u_{yy} = u_x + 6y$$

Problem 4 (10 points). (Section 2.2) Solve the initial value problem

$$\mathbf{u}_t + \begin{bmatrix} -4 & -6 \\ 3 & 5 \end{bmatrix} \mathbf{u}_x = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad \mathbf{u}(x, 0) = \begin{bmatrix} x \\ 0 \end{bmatrix}$$

Problem 5 (10 points). (Section 2.2) Find the general solution to the linear system

$$\begin{aligned} u_t + v_x &= u \\ v_t + u_x &= v \end{aligned}$$

[1] All problems are from McOwen's book