

CALCULUS III – MTH 253

CALCULUS I & II REVIEW

Due Wednesday, June 24

Name: _____

Show all necessary work to justify your answer.

- (1) Evaluate the following integral where $a > 0$. (Answer depends on a .)

$$\int_0^{\infty} e^{-ax} dx$$

- (2) Evaluate the following integral in which b is arbitrary. (Answer depends on b .)

$$\int_0^{\pi/2} \sqrt{b + \sin x} \cos x dx$$

- (3) Compute the following limits

(a) $\lim_{x \rightarrow \infty} \frac{x^3}{e^{x^2}}$

(b) $\lim_{x \rightarrow \infty} \frac{\sqrt{x}}{\ln x}$

- (4) Consider the following functions: $f(x) = 3x^2 - 2x + 2$ and $g(x) = x^2 + x + 1$. Prove that $f(x) \geq g(x)$ for all $x \geq 1$ as follows.

(a) Show that $f(1) - g(1) = 0$.

(b) Show that $f'(x) - g'(x) > 0$ for $x \geq 1$.

(c) Explain why items (a) and (b) above lead to the desired conclusion.