

Mth 413/513  
Spring 2009

## Topics for the final exam

The exam will include examples and counterexamples, and some exercises of the type and level discussed in the homework assignments and class problems.

You should know the proofs of the following theorems, not because you will be asked to reproduce them directly but rather because they provide useful techniques.

- Cauchy's inequality (Theorem 9.1)
- Closest point theorem (Theorem 9.2)
- Equivalences for an orthonormal set to be a basis (Theorem 9.6)
- Riesz representation theorem for functionals on a Hilbert space (Theorem 9.8)
- Hölder's inequality (Theorem 9.9)
- Minkowski's inequality (Theorem 9.10)
- Riesz's theorem on the completeness of  $L^p$  (Theorem 9.11)
- Riesz Representation theorem in  $L^p$  (Theorem 9.12)
- Hahn-Banach theorem and its corollaries (Theorem 10.2 and 10.3 and Corollary 10.2)
- Open Mapping Theorem (Theorem 10.5)
- Closed Graph Theorem (Theorem 10.6)
- Theorem 11.1 about the Fourier coefficients
- Proposition 11.2 on properties of the Dirichlet kernel.
- Dirichlet's Theorem (Theorem 11.3)

You should be familiar with the definitions of all objects discussed. Collect examples and counterexamples to illustrate the various definitions and theorems. You should be able to calculate some Fourier series and analyze their convergence.