< HW #2 >

Section 4.6 Exercises.

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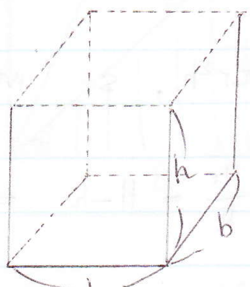

b: length of the base

h: length of the height.

$$\text{Volume} = 32,000$$
$$= b^2 h$$
$$\Rightarrow h = 32,000/b^2.$$

The surface area of the open box is

$$S = b^2 + 4hb = b^2 + 4\,(32,000/b^2)\cdot b = b^2 + 4\,(32,000)/b.$$

So, $S'(b) = 2b - 4\,(32,000)/b^2 = 2(b^3 - 64,000)\,b^{-2} = 0$

$$\Rightarrow b = \sqrt[3]{64,000} = 40.$$

This gives an absolute min. since $S'(b) < 0$ if $0 < b < 40$ and $S'(b) > 0$ if $b > 40$. The box should be $40 \times 40 \times 20$.

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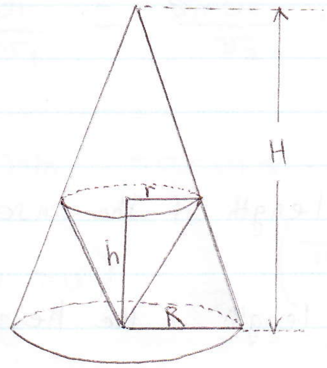

By similar triangles, $\frac{H}{R} = \frac{H-h}{r}$ ..... (1).

The volume of the inner cone is

$V = \frac{1}{3}\pi r^2 h$, so we'll solve (1) for h.

From (1), $\frac{Hr}{R} = H - h \Rightarrow$

$$h = H - \frac{Hr}{R}$$

$$= \frac{HR - Hr}{R} = \frac{H}{R}(R-r) \text{ ..... (2).}$$

Thus, $V(r) = \frac{\pi}{3} r^2 \cdot \frac{H}{R}(R-r) = \frac{\pi}{3}\frac{H}{R}(Rr^2 - r^3)$

$\Rightarrow \quad V'(r) = \frac{\pi}{3}\frac{H}{R}(2Rr - 3r^2) = \frac{\pi H}{3R} r \cdot (2R - 3r)$.

$V'(r) = 0 \Rightarrow r = 0$ or $2R = 3r$.

$\Rightarrow r = \frac{2}{3}R$ and from (2), $h = \frac{H}{R}(R - \frac{2}{3}R) = \frac{H}{R}(\frac{1}{3}R) = \frac{1}{3}H$.

$V'(r)$ changes from positive to negative at $r = \frac{2}{3}R$, so the inner cone has a max. vol. of $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(\frac{2}{3}R)^2 \cdot \frac{1}{3}H$

$$= \frac{4}{27} \frac{1}{3}\pi R^2 H,$$

which is approx. 15% of the vol. of the larger cone.

$$\sqrt{x^2-(8-x)^2} = \sqrt{x^2-64+16x-x^2} = 4\sqrt{x-4}$$

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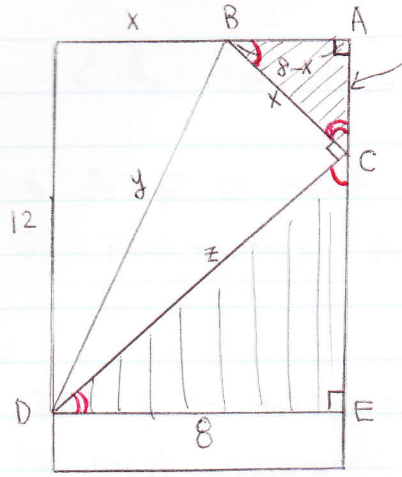

$y^2 = x^2 + z^2$, but

ΔBAC and ΔCED are similar,

so $\dfrac{z}{8} = \dfrac{x}{4\sqrt{x-4}}$

$\Rightarrow z = 2x/\sqrt{x-4}$.

Thus, we minimize $f(x) = y^2 = x^2 + 4x^2/(x-4) = x^3/(x-4)$, $4 < x \le 8$.

$$f'(x) = \frac{(x-4)(3x^2) - x^3}{(x-4)^2} = \frac{x^2(3(x-4)-x)}{(x-4)^2} = \frac{2x^2(x-6)}{(x-4)^2} = 0$$

when $x=6$. $f'(x) < 0$ when $x < 6$ and $f'(x) > 0$ when $x > 6$,

So the min. occurs when $x = 6$ in.

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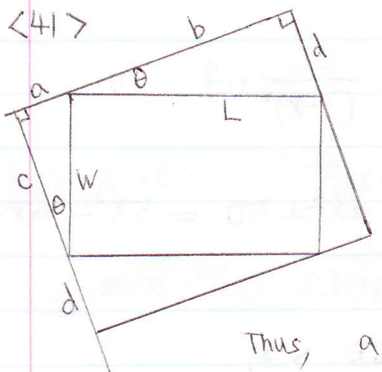

In the smaller triangle w/ sides a and c and hypotenuse W,

$\sin\theta = \dfrac{a}{W}$ and $\cos\theta = \dfrac{c}{W}$. In the Δ with sides b and d and hypotenuse L, $\sin\theta = \dfrac{d}{L}$ and $\cos\theta = \dfrac{b}{L}$.

Thus, $a = W\sin\theta$, $c = W\cos\theta$, $d = L\sin\theta$, $b = L\cos\theta$, so the ~~angle~~ area of the circumscribed rectangle is

$$A(\theta) = (a+b)(c+d) = (W\sin\theta + L\cos\theta)(W\cos\theta + L\sin\theta)$$

$$= W^2\sin\theta\cos\theta + WL\sin^2\theta + LW\cos^2\theta + L^2\sin\theta\cos\theta$$

$$= LW\sin^2\theta + LW\cos^2\theta + (L^2+W^2)\sin\theta\cos\theta$$

$$= LW(\sin^2\theta + \cos^2\theta) + (L^2+W^2)\tfrac{1}{2}\cdot 2\cdot\sin\theta\cos\theta$$

$$= LW + \tfrac{1}{2}(L^2+W^2)\sin 2\theta, \quad 0 \le \theta \le \tfrac{\pi}{2}.$$

Max. occurs when $\sin 2\theta$ is maximized, which occurs $\sin 2\theta = 1$, $\Rightarrow \theta = \frac{\pi}{4}$.

So, the max. Area is $A(\frac{\pi}{4}) = LW + \frac{1}{2}(L^2+W^2) = \frac{1}{2}(L+W)^2$.

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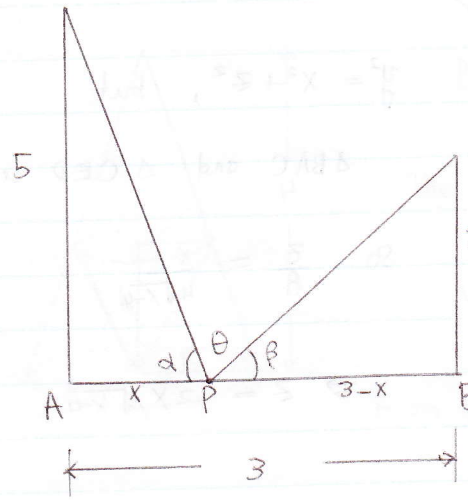

From the figure, $\tan\alpha = 5/x$ and $\tan\beta = 2/(3-x)$.

Since $\alpha+\beta+\theta = 180° = \pi$, $\theta = \pi - \tan^{-1}(5/x) - \tan^{-1}(2/(3-x))$.

$$\Rightarrow \frac{d\theta}{dx} = -\frac{1}{1+\left(\frac{5}{x}\right)^2}\left(-\frac{5}{x^2}\right) - \frac{1}{1+\left(\frac{2}{(3-x)}\right)^2}\left(\frac{2}{(3-x)^2}\right).$$

$$= \frac{x^2}{x^2+25}\cdot\frac{5}{x^2} - \frac{(3-x)^2}{(3-x)^2+4}\cdot\frac{2}{(3-x)^2}.$$

$$\frac{d\theta}{dx} = 0 \Rightarrow \frac{5}{x^2+25} = \frac{2}{x^2-6x+13} \Rightarrow 2x^2+50 = 5x^2-30x+65$$

$$\Rightarrow x = 5 \pm 2\sqrt{5}, \text{ but } 5+2\sqrt{5} > 3.$$

$$\Rightarrow x = 5-2\sqrt{5}.$$

$\frac{d\theta}{dx} > 0$ for $x < 5-2\sqrt{5}$ and $\frac{d\theta}{dx} < 0$ for $x > 5-2\sqrt{5}$,

so θ is maximized when $|AP| = x = 5-2\sqrt{5} \approx .53$.