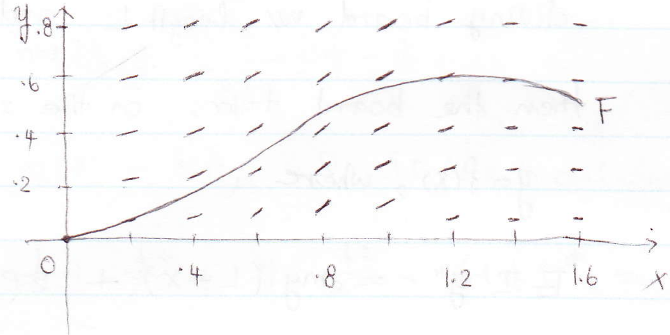
< HW #3 > for Section 4.9.

<37>



<44>

1st ball is thrown w/ a speed of 48 ft/s and the other is thrown a second later w/ a speed of 24 ft/s. Do the balls ever pass each other?

Ans). For the 1st ball, $S_1(t) = -16t^2 + 48t + 432$ from the example we had.

For the 2nd ball, $a(t) = -32$.

$$\Rightarrow v(t) = -32t + C, \text{ but}$$

$$v(1) = -32(1) + C = 24$$

$$\Rightarrow C = 56.$$

$$\Rightarrow S_2(t) = -16t^2 + 56t + D, \text{ but}$$

$$S_2(1) = -16(1)^2 + 56(1) + D = 432,$$

$$\Rightarrow D = 392, \text{ and so } S_2(t) = -16t^2 + 56t + 392.$$

$$S_1(t) = S_2(t) \Rightarrow -16t^2 + 48t + 432 = -16t^2 + 56t + 392$$

$$\Rightarrow 8t = 40 \Leftrightarrow t = 5s.$$

<55> A high-speed bullet train accelerates and decelerates at the rate of $4\ ft/s^2$. Its max. cruising speed is 90 mi/h.

(a) what is the max. dist. the train can travel if it accelerates from rest until it reaches its cruising speed and then runs at that speed for 15 minutes?

Ans). $90\ mi/h = 90 \times \frac{5280}{3600}\ ft/s = 132\ ft/s$.

Then $a(t) = 4\ ft/s^2 \Rightarrow v(t) = 4t + C$, but $v(0) = 0$

$\Rightarrow C = 0$. Now, $4t = 132$ when $t = \frac{132}{4} = 33\ s$.

So, it takes 33 s to reach 132 ft/s. Therefore, taking $s(0) = 0$,

we have $s(t) = 2t^2$, $0 \le t \le 33$.

So, $s(33) = 2178$ ft, 15 minutes $= 15(60) = 900\ s$,

So for $33 < t \le 933$ we have $v(t) = 132$ ft/s.

$\Rightarrow s(933) = 132(900) + 2178 = 120{,}978$ ft $= 22.9125$ mi.

(b) S'pose that the train starts from rest and must come to a complete stop in 15 minutes. What is the max. distance it can travel under these conditions?

Ans) As in part (a), the train accelerates for 33 s and travel 2178 ft while doing so. Similarly, it decelerates for 33 s and travels 2178 ft at the end of its trip. During the remaining $900 - 66 = 834$ s it travels at 132 ft/s, so the distance traveled is $132 * 834 = 110{,}088$ ft. Thus, the total distance is

$2178 + 110{,}088 + 2178 = 114{,}444$ ft $= 21.675$ mi.

c) Find the min. time that the train takes to travel between two consecutive stations that are 45 miles apart.

Ans) 45 mi = 45(5280) = 237,600 ft. Subtract 2(2178) to take care of the speeding up and slowing down, and we have

233,244 ft/s for a trip of 233,244/132 = 1767 s at
ft at 132

90 mi/h. The total time is 1767 + 2(33) = 1833 s

= 30 min 33 s = 30.55 min

d) The trip from station to the next takes 37.5 minutes. How far apart the stations?

Ans). 37.5(60) = 2250 s.

2250 − 2(33) = 2184 s at max. speed.

2184 (132) + 2(2178) = 292,644 ft or

292,644/5280 = 55.425 mi.

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