< HW #4: section 5.2 >

<42> $\int_1^5 f(x)dx = 12$ and $\int_4^5 f(x)dx = 3.6$. Find $\int_1^4 f(x)dx$.

Ans) $\int_1^5 f(x)dx = \int_1^4 f(x)dx + \int_4^5 f(x)dx$

$\int_1^5 f(x)dx = 12$

$$\Rightarrow \int_1^4 f(x)dx = 12 - 3.6 = 8.4$$

<22> $A = \int_1^4 (x^2 + 2x - 5)\, dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$, where x_i's are right endpoints and $\Delta x = \frac{4-1}{n} = \frac{3}{n}$.

$$A = \lim_{n\to\infty} \frac{3}{n}\sum_{i=1}^{n} f\left(1+\frac{3i}{n}\right) = \lim_{n\to\infty} \frac{3}{n}\cdot\sum_{i=1}^{n}\left(\left(1+\frac{3i}{n}\right)^2 + 2\left(1+\frac{3i}{n}\right) - 5\right)$$

$$= \lim_{n\to\infty} \frac{3}{n}\cdot\sum_{i=1}^{n}\left(1 + \frac{6i}{n} + \frac{9}{n^2}i^2 + 2 + \frac{6i}{n} - 5\right)$$

$$= \lim_{n\to\infty} \frac{3}{n}\left(-2n + \frac{12}{n}\cdot\frac{n(n+1)}{2} + \frac{9}{n^2}\cdot\frac{n(n+1)(2n+1)}{6}\right)$$

$$= \lim_{n\to\infty}\left(-6 + 18\,\frac{n(n+1)}{n^2} + \frac{9}{2}\,\frac{n(n+1)(2n+1)}{n^3}\right)$$

$$= -6 + 18 + \frac{9}{2} * 2 = -6 + 18 + 9 = 21.$$