KEY

Quiz IV for Math252

August 11, 2009

Name:

Instructions: There are two questions. You are given 45 minutes. Show all your work to get partial credits. Good luck!

1. The manager of a fast-food restaurant determines that the average time that her customer's waiting time for service, X, has the following probability density function:

$$f(x) = \begin{cases} 0, & x \le 0 \\ ce^{-cx}, & x > 0 \end{cases}$$

(a) If the average waiting time for service is 2.5 minutes, what should the constant c be?

$$\mu = \int_{-\infty}^{\infty} x f(x)\,dx = \int_{0}^{\infty} x \cdot c e^{-cx}\,dx + \int_{-\infty}^{0} x \cdot 0\,dx$$

$$= \int_0^{\infty} cx e^{-cx}\,dx = 2.5$$

Solve for c.

Let $u = x$ and $dv = ce^{-cx}dx$.

$$\int_0^{\infty} cx e^{-cx}\,dx = \left[x \cdot (-e^{-cx})\right]_0^{\infty} - \int_0^{\infty} 1 \cdot (-e^{-cx})\,dx$$

$$= \int_0^{\infty} e^{-cx}\,dx = -\frac{1}{c}\left[e^{-cx}\right]_0^{\infty}$$

$$= \frac{1}{c}$$

$$\Rightarrow \frac{1}{c} = 2.5 \Rightarrow c = \frac{1}{2.5} = .4$$

(b) Find the probability that a customer is served within the first 2 minutes.

$$P(0 < X \leq 2) = \int_0^2 .4 e^{-.4x}\, dx$$

$$= \left[-e^{-.4x} \right]_0^2 = 1 - e^{-.8}$$

$$= .5507$$

(c) The manager wants to advertise that anybody who isn't served within a certain number of minutes gets a free hamburger. But she doesn't want to give away free hamburgers to more than 2% of her customers. What should the advertisement say?

$$P(0 < X < x_0) = .98$$

$$LHS = \int_0^{x_0} .4 e^{-.4x}\, dx = \left[-e^{-.4x} \right]_0^{x_0}$$

$$= 1 - e^{-.4x_0} = .98 \quad RHS$$

$$\Rightarrow \quad e^{-.4x_0} = .02$$

$$\Rightarrow \quad -.4 x_0 = \ln(.02)$$

$$\Rightarrow \quad x_0 = -\frac{1}{.4} \ln(.02) = 9.78$$

She should say anybody who is not served in 10 minutes gets a free hamburger.