STAT244NB Introduction to Probability and Statistics II

Quiz # 2 (7/16/09) Key

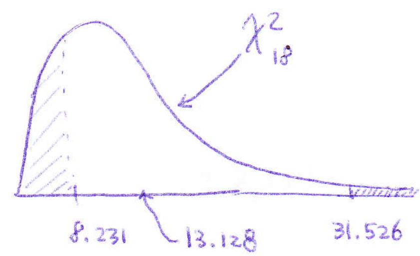
1. In 1990, the average pH level of the rain in Pierce ~~County,~~ Washington, was 5.03. A biologist claims that the acidity of rain has increased. (This would mean that the pH level of the rain has decreased.) From a random sample of 19 rain dates in 2000, she obtains the following data:

5.08, 4.66, 4.70, 4.87, 4.78, 5.00, 4.50, 4.73, 4.79, 4.65, 4.91, 5.07, 5.03, 4.78, 4.77, 4.60, 4.73, 5.05, 4.70. (Source: National Atmospheric Deposition Program) For your information, $\bar{x} = 4.8105$ and s = 0.1708. Note: $z_{.01} = 2.33$; $z_{.05} = 1.645$; $z_{.025} = 1.96$; $t_{18,.01} = 2.552$; $t_{18,.05} = 1.734$; $\chi^2_{18,.975} = 8.231$; $\chi^2_{18,.025} = 31.526$

(a) Before you test the hypothesis on the population mean, test the assumption that $\sigma = 0.2$ at $\alpha = 0.05$ level of significance, assuming that pH level is normally distributed.

$H_0: \sigma^2 = .2^2$ vs. $H_1: \sigma^2 \neq .2^2$

Test Stat $\chi^2 = \dfrac{(19-1)\cdot .1708^2}{.2^2} = 13.128$



C.V.'s: $\chi^2_{18,.025} = 31.526$; $\chi^2_{18,.975} = 8.231$

Since our test stat 13.128 is between 8.231 and 31.526, we do not reject Ho.

(b) Test the hypothesis, assuming that pH level is normally distributed with $\sigma = 0.2$ at the $\alpha = 0.01$ level of significance. Use the p-value method.

① $H_0: \mu \geq 5.03$ vs. $H_1: \mu < 5.03$, where μ stands for the (mean) average pH level of the rain.

② Test stat $Z = \dfrac{4.8105 - 5.03}{.2/\sqrt{19}} = -4.784$

③ p-value $= P(Z < -4.784) < P(Z < -2.33) = .01 = \alpha$.

Since the p-value is $< \alpha = .01$, we reject the Ho and conclude that the pH level of the rain has decreased. In other words, the acidity of rain has increased.

(c) Compute the power of your test at $\mu = 4.5$ in part (b).

① $H_0: \mu \geq 5.03$ vs. $H_1: \mu < 5.03$

Test stat. $Z = \dfrac{\bar{X} - 5.03}{.2/\sqrt{19}}$

② Decision rule: Reject Ho if $Z < -Z_\alpha = -2.33$.

$Z = \dfrac{\bar{X} - 5.03}{.2/\sqrt{19}} < -2.33 \Rightarrow \bar{X} < 5.03 - 2.33\left(\dfrac{.2}{\sqrt{19}}\right) = 4.923$

③ Power at $\mu = 4.5 = P(\bar{X} < 4.923 \mid \mu = 4.5)$

$= P\left(\dfrac{\bar{X} - 4.5}{.2/\sqrt{19}} < \dfrac{4.923 - 4.5}{.2/\sqrt{19}}\right)$

$= P(Z < 9.371) > P(Z < Z_{.01} = 2.33) = .99$

Thus, power at $\mu = 4.5$ is greater than .99 and almost identical to 1, which means the z-test can reject the null (5.03) with almost 100% accuracy when the true mean is 4.5.

(d) Repeat (b) by constructing one-tailed confidence interval.

Hint: Use $(-\infty, \bar{x} + z_\alpha \dfrac{\sigma}{\sqrt{n}})$ as the one-tailed confidence interval. If the μ_0 belongs to the interval, do not reject H_0. Otherwise, reject H_0.

$\left(-\infty, 4.8105 + 2.33 \dfrac{.2}{\sqrt{19}}\right) = (-\infty, 4.917)$.

Since 5.03 does not belong to the interval, reject Ho, and conclude that the pH level of the rain has decreased.