Stat 451 Assignment #2 Total 23

1) a) Compute $\overline{X}$, median, and mode

$\overline{X}$ = 756.217

Median = 774 3

Mode = 748 + 796

b) Compute R, IQR, Standard Deviation, and MAD

range = 1051 - 573 = 478

25th percentile = 23(.25) = 5.75 so 6th position

Q_1 = 696

75th percentile = 23(.25) = 17.75 so 18th position 4

Q_3 = 809

IQR = $Q_3 - Q_1$ = 809 - 696 = 113

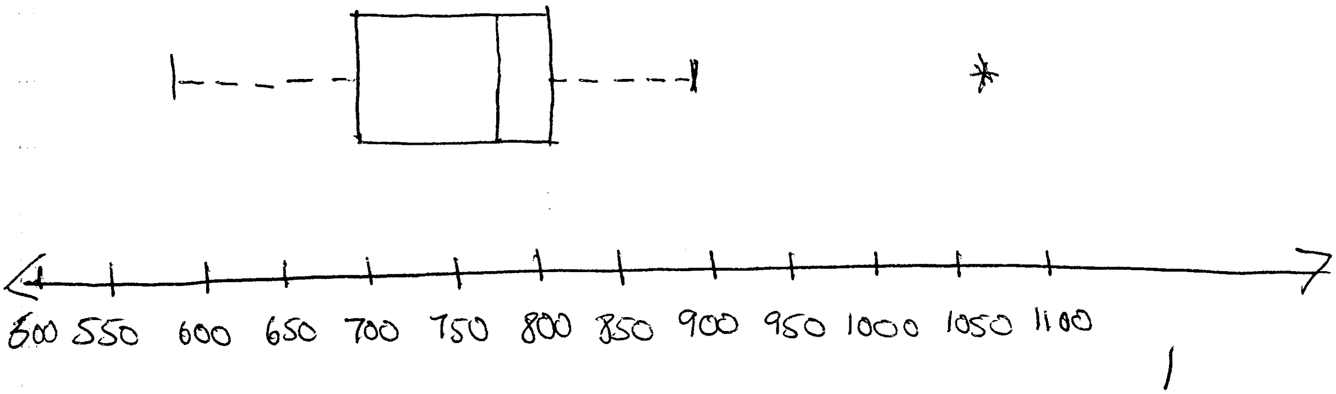
S = 107.115

$$MAD = \sum_{i=1}^{n} \frac{|x_i - \bar{x}|}{n} = 77.6677$$

c) Provide 5-number summary table 1

	774	
696		809
573		1051

1d) Construct a Box-plot



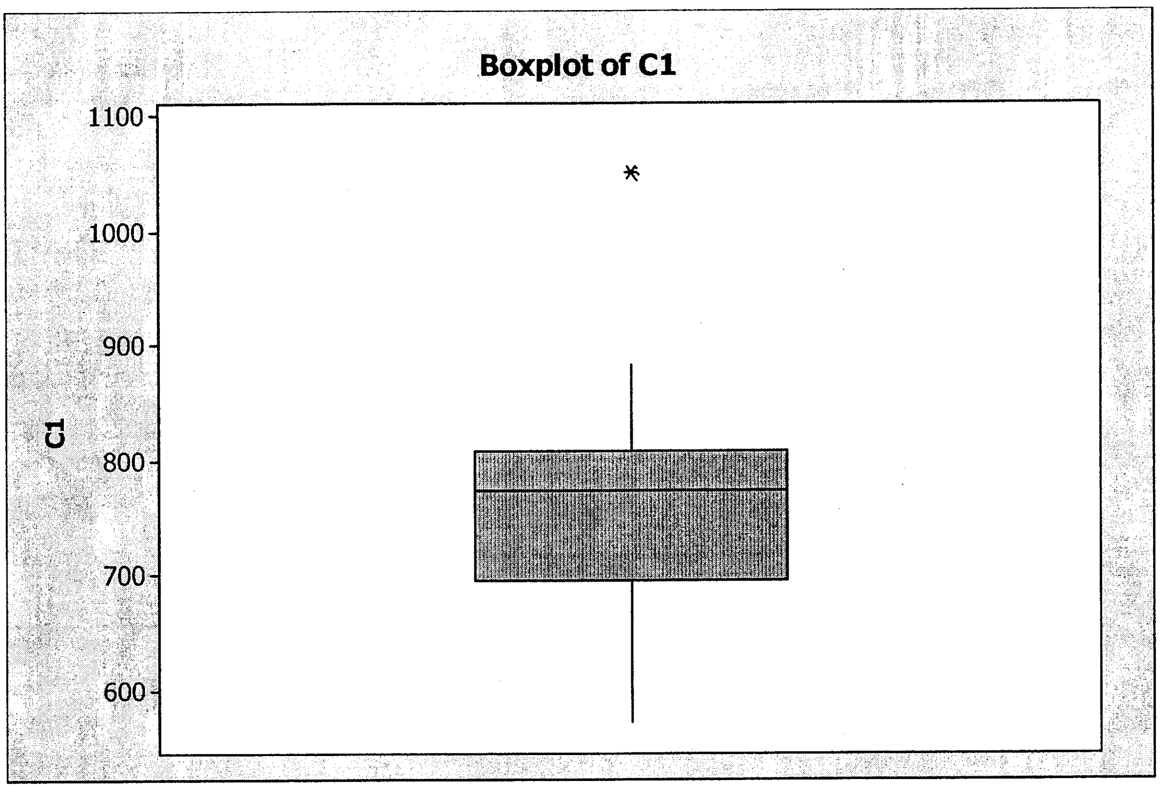
2.

Descriptive Statistics: C1

Variable	N	N*	Mean	SE Mean	StDev	Sum of Squares	Minimum	Q1	Median
C1	23	0	756.2	22.3	107.1	13405307.0	573.0	696.0	774.0

Variable	Q3	Maximum	Range	IQR	Mode	N for Mode
C1	809.0	1051.0	478.0	113.0	748, 796	2

MAD=77.667



2

3) $n = 200 \quad \bar{x} = 50 \quad s = 5$

a) $P(30 < x < 70) \quad 70 = \bar{x} + ks = 50 + k5 \Rightarrow 20 = k5 \Rightarrow \boxed{k = 4}$

$$1 - \frac{1}{k^2} = 1 - \frac{1}{4^2} = 15/16 = 93.75\%$$

$.9375 \times 200 = 187.5$

So at least 188 values fall within this region

b) $50 - 40 = 10 \quad ks = 10 \quad k = 2$

$$\frac{1}{k^2} = \frac{1}{2^2} = \frac{1}{4}$$

so at most 50 values will fall ~~in~~ outside this region

$.25 \times 200 = 50$

√

68) a) $\frac{d}{dc}\left(\sum (x_i - c)^2\right) = \sum \frac{d}{dc}(x_i - c)^2 = -2\sum (x_i - c) = 0$

$\Rightarrow \sum (x_i - c) = 0$

$\Rightarrow \sum x_i - \sum c = 0 \Rightarrow \sum x_i - nc = 0$

$\Rightarrow nc = \sum x_i \Rightarrow c = \frac{\sum x_i}{n} = \bar{x}$ 1

b) $\sum (x_i - \bar{x})^2 < \sum (x_i - \mu)^2$ 1

69) a) $\bar{y} = \frac{\sum y_i}{n} = \frac{\sum (ax_i + b)}{n} = \frac{a\sum x_i + b}{n} = a\bar{x} + b$

$s_y^2 = \frac{\sum (y_i - \bar{y})^2}{n-1} = \frac{\sum (ax_i + b - (a\bar{x} + b))^2}{n-1}$ 1

$\therefore S_y = |a| S_x$ 1

$= \frac{\sum (ax_i - a\bar{x})^2}{n-1} = \frac{a^2 \sum (x_i - \bar{x})^2}{n-1} = a^2 s_x^2$

b) $\frac{9}{5}c + 32 \Rightarrow \frac{9}{5}(87.3) + 32 = 189.14\ {}^\circ F$ 1

$\frac{9}{5}c + 32 \Rightarrow \frac{9}{5}(1.04) = 1.87$ 1

78.

a. Since the constant $\bar{x}$ is subtracted from each x value to obtain each y value, and addition or subtraction of a constant doesn't affect variability, $s_y^2 = s_x^2$ and $s_y = s_x$

b. Let $c = 1/s$, where s is the sample standard deviation of the x's and also (by a) of the y's. Then $s_z = cs_y = (1/s)s = 1$, and $s_z^2 = 1$. That is, the "standardized" quantities $z_1, \ldots, z_n$ have a sample variance and standard deviation of 1.

78)
a) $y_i = x_i - \bar{x} \qquad \bar{y} = \frac{\sum y_i}{n} = \frac{\sum (x_i - \bar{x})}{n}$

$$= \frac{\sum x_i - n\bar{x}}{n} = \frac{n\bar{x} - n\bar{x}}{n} = 0$$ 1

b) $z_i = \frac{(x_i - \bar{x})}{s} \qquad \bar{z} = \frac{\sum z_i}{n} = \frac{\sum (x_i - \bar{x})}{s \, n}$

$$= \frac{n\left[\sum x_i - n\bar{x}\right]}{sn} = \frac{n\bar{x} - n\bar{x}}{s} = 0$$ 1

79) a) $\sum_{i=1}^{n+1} x_i \qquad \sum_{i=1}^{n} x_i + x_{n+1} = n\bar{x}_n + x_{n+1}$

$$\bar{x}_{n+1} = \frac{n\bar{x}_n + x_{n+1}}{n+1}$$

b) $n s_{n+1}^2 = \sum_{i=1}^{n+1} (x_i - \bar{x}_{n+1})^2 = \sum_{i=1}^{n+1} x_i^2 - (n+1)\bar{x}_{n+1}^2$

$$= \sum_{i=1}^{n} x_i^2 - n\bar{x}_n^2 + x_{n+1}^2 + n\bar{x}_n^2 - (n+1)\bar{x}_{n+1}^2$$

$$= (n-1)s_n^2 + \left\{ x_{n+1}^2 + n\bar{x}_n^2 - (n+1)\bar{x}_{n+1}^2 \right\}$$

substitute $\bar{x}_{n+1}$ from a

$$= (n-1)s_n^2 + \left\{ x_{n+1}^2 + n\bar{x}_n^2 - (n+1)\left(\frac{n\bar{x}_n + x_{n+1}}{n+1}\right)^2 \right\}$$

$$\Rightarrow (n-1)s_n^2 + \left\{ \frac{x_{n+1}^2(n+1) + n\bar{x}_n^2(n+1) - n^2\bar{x}_n^2 - 2n\bar{x}_n x_{n+1} - x_{n+1}^2}{n+1} \right.$$

$$\Rightarrow (n-1)s_n^2 + \left\{ \frac{n x_{n+1}^2 + \cancel{x_{n+1}^2} + \cancel{n^2\bar{x}_n^2} + n\bar{x}_n^2 - \cancel{n^2\bar{x}_n^2} - 2n\bar{x}_n x_{n+1} - \cancel{x_{n+1}^2}}{n+1} \right.$$

$$= (n-1)s_n^2 + \frac{n(x_{n+1}^2 - 2\bar{x}_n\bar{x}_{n+1} + \bar{x}_n^2)}{n+1}$$

$$= (n-1)s_n^2 + \frac{n(x_{n+1} - \bar{x}_n)^2}{n+1} \quad ✓$$

79) c) $\bar{x}_{n+1} = \frac{15(12.58)+11.8}{16} = \frac{200.5}{16} = 12.53$

$$s_{n+1}^2 = \frac{n-1}{n}(s_n^2) + \frac{(x_{n+1}-\bar{x}_n)^2}{n+1} = \frac{14}{15}(.512^2) + \frac{(11.8-12.58)^2}{16}$$

$= .245 + .038 = .238$ So $s_{n+1} = \sqrt{.238} = .532$