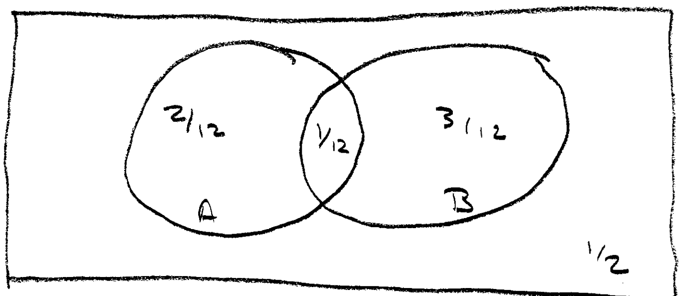
Stat 451 Assignment #3

1) Suppose A and B are events $P(A) = 1/4$ $P(B) = 1/3$
$P(A \cup B) = 1/2$, Find

a) $P(A \cap B)$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$\Rightarrow P(A \cap B) = P(A) + P(B) - P(A \cup B)$$
$$= 1/4 + 1/3 - \tfrac{1}{2}$$
$$= 3/12 + 4/12 - 6/12$$
$$= \tfrac{1}{12}$$

so we have



b) $P(A \cap B') = 2/12 = 1/6$

c) $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{1/12}{1/3} = \dfrac{1}{4}$

d) Are the events mutually exclusive?
No because $P(A \cap B) \neq \emptyset$

e) Are A + B independent events
Yes because $P(A \mid B) = P(A)$

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2) A fair dime is tossed three times. Find the probability that

(a) all three tosses come up the same

$$P(HHH) + P(TTT) = (1/2)(1/2)(1/2) + (1/2)(1/2)(1/2)$$
$$= 1/8 + \frac{1}{8}$$
$$= 1/4$$

(b) at least one head comes up

P(At least one head comes up)
$= 1 -$ P(no heads come up)
$= 1 - P(TTT) = 1 - (1/2)(1/2)(1/2) = 1 - 1/8 = 7/8$

(c) A tail results on the second toss

$\boxed{1/2}$

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3) $P(A) = .4$ $P(A|B) = .6$ $P(B|A) = .3$

(a) Find $P(A \cap B)$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} \Rightarrow P(A \cap B) = P(A)\,P(B|A)$$

$$= .4\,(.3)$$

$$= \boxed{.12}$$

(b) Find $P(B)$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \Rightarrow P(B) = \frac{P(A \cap B)}{P(A|B)}$$

$$= \frac{.12}{.6}$$

$$= \boxed{.2}$$

c) Are the events A and B independent

No

$$P(A|B) \neq P(A)$$

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4) P(Experiences Heart Disease | smokes)

$$= \frac{P(\text{Heart Disease} \cap \text{smokes})}{P(\text{smokes})}$$

$$= \frac{.10}{.30} = .333$$

5) #91

(a) $P(\text{line } 1) = \frac{500}{1500} = 1/3$

$$P(\text{Crack}) = \frac{(.50)(500) + .44(400) + .40(600)}{1500} = \frac{666}{1500} = .444$$

(b) $P(\text{blemish} \mid \text{line } 1) = .15$

(c) $$P(\text{Surface defect}) = \frac{(.10)(500) + .08(400) + .15(600)}{1500} = \frac{172}{1500}$$

$$P(\text{Line } 1 \cap \text{Surface Defect}) = \frac{.10(500)}{1500} = \frac{50}{1500}$$

$$P(\text{Line } 1 \mid \text{Surface Defect}) = \frac{P(\text{Line } 1 \cap \text{Surface Defect})}{P(\text{Surface Defect})} = \frac{\frac{50}{1500}}{\frac{172}{1500}} = \frac{50}{172}$$

$$= \boxed{.291}$$

#6 Ex. 33, p.66

15 players 9 positions

(a) LF CF RF 3rd SS 2nd 1st P C

$15 \times 14 \times 13 \times 12 \times 11 \times 10 \times 9 \times 8 \times 7$

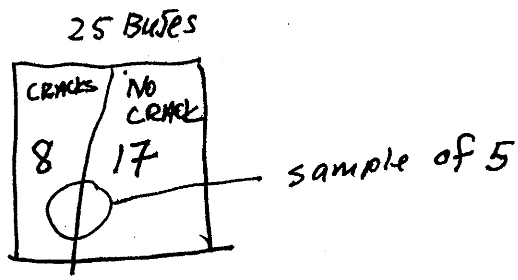
$$= {}_{15}P_9 = \frac{15!}{(15-9)!} = \frac{15!}{6!} = 1{,}816{,}214{,}400$$

(b) ${}_{15}P_9 \times 9!$

(c) $\binom{5}{3} \times \binom{10}{6} = {}_5C_3 \times {}_{10}C_6 = 2100$

$$= \frac{5!}{3!2!} \times \frac{10!}{6!4!} =$$

#7 Ex. 34, p.66



(a) ${}_{25}C_5 = \frac{25!}{5!20!} = 53{,}130$

(b) ${}_8C_4 \times {}_{17}C_1 = 70 \times 17 = 1{,}190$

(c) P(exactly 4 of the 5 buses have cracks) $= \frac{(b)}{(a)} = \frac{1190}{53130}$

$\approx .0224$

(d) at least 4 $\Rightarrow$ 4 or 5 cracks

$P(4) + P(5) = .0224 + \frac{{}_8C_5}{{}_{25}C_5} = .0224 + \frac{56}{(a)}$

$= .0224 + .0011 = .02345$

#8 EX. 35, P. 66

45 workers

20 DS, 15 SS, 10 GS

a

$_{20}C_6 = 38760$

$_{45}C_6 = 8{,}145{,}060$

$\boxed{\text{Prob} \approx .0048}$

b

A = all 6 from same shift

$= \underbrace{\{\text{all } \cancel{6}\text{ from DS}\}}_{A_1} \cup \underbrace{\{\text{all 6 from SS}\}}_{A_2} \cup + \underbrace{\{\text{all 6 from GS}\}}_{A_3}$

$$P(A) = P(A_1) + P(A_2) + P(A_3)$$

$$= \frac{_{20}C_6 + {_{15}C_6} + {_{10}C_6}}{_{45}C_6} = \frac{43975}{8145060} \approx \boxed{.0054}$$

c

let A = { at least two different shifts are represented in the 6 workers chosen }

= { two or three shifts }

Then A' = only one shift. $P(A')$ is given in part b

$\therefore \quad P(A) = 1 - P(A') = 1 - .0054 = .9946$

#8 d Let DS' = day shift unrepresented,
SS' = swing " "
GS' = graveyard shift ".

Let B = event at least one shift is unrepresented.

Then $B = DS'$ or SS' or GS'
$= DS' \cup SS' \cup GS'$

We use the formula in the blue box on page 56 in book:

$$P(B) = P(DS') + P(SS') + P(GS') - P(DS' \cap SS') - P(DS' \cap GS') - P(SS' \cap GS')$$

$$= \frac{{}_{25}C_6 + {}_{30}C_6 + {}_{35}C_6 - {}_{10}C_6 - {}_{15}C_6 - {}_{20}C_6}{{}_{45}C_6}$$

$$= \frac{2,350,060}{8,145,060} = .2885$$

The long way $\frown$

#8

(d) Let B = at least one shift is unrepresented

Then B' = none are missing; i.e. all three are represented

20 DS	15 SS	10 GS	Count
1	4	1	$_{20}C_1 \times {}_{15}C_4 \times {}_{10}C_1 = 273,000$
or	1	4	$_{20}C_1 \times {}_{15}C_1 \times {}_{10}C_4 = 63,000$
or	3	2	$_{20}C_1 \times {}_{15}C_3 \times {}_{10}C_2 = 409,500$
or	2	3	$_{20}C_1 \times {}_{15}C_2 \times {}_{10}C_3 = 252,000$
OR			
2	3	1	$_{20}C_2 \times {}_{15}C_3 \times {}_{10}C_1 = 864,500$
or	1	3	$_{20}C_2 \times {}_{15}C_1 \times {}_{10}C_3 = 342,000$
or	2	2	$_{20}C_2 \times {}_{15}C_2 \times {}_{10}C_2 = 897,750$
or			
3	2	1	$_{20}C_3 \times {}_{15}C_2 \times {}_{10}C_1 = 1,197,000$
or	1	2	$_{20}C_3 \times {}_{15}C_1 \times {}_{10}C_2 = 769,500$
or			
4	1	1	$_{20}C_4 \times {}_{15}C_1 \times {}_{10}C_1 = 726,750$

total # in B' = 5,795,000

$$P(B) = 1 - P(B') = 1 - \frac{5,795,000}{8,145,060} = \boxed{.2885}$$

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9) $P(\text{Mr Adams}) = .3$ $\quad P[\text{fees Increased} \mid \text{Mr Adams}] = .8$

$P(\text{Mr Brown}) = .5$ $\quad P[\text{fees Increased} \mid \text{Mr Brown}] = .1$

$P(\text{Mr Cooper}) = .2$ $\quad P[\text{fees Increased} \mid \text{Mr Cooper}] = .4$

$$P(\text{Mr Cooper} \mid \text{Fees Increased})$$

$$= \frac{P[\text{Fees Increased} \mid \text{Mr Cooper}]\, P[\text{Mr Cooper}]}{P[\text{Fees Increased}]}$$

$$= \frac{(.4)(.2)}{(.4)(.2) + (.8)(.3) + (.1)(.5)} = \frac{.08}{.37} = \boxed{.2162}$$

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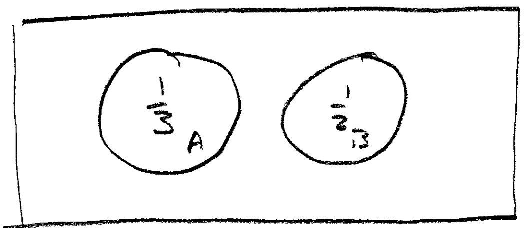
10) Consider the two events A and B such that $P(A) = 1/3$ & $P(B) = 1/2$

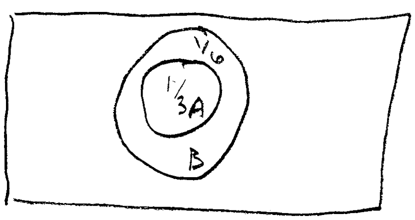
Under each of the following conditions determine $P(A' \cap B)$

(a) A & B are disjoint



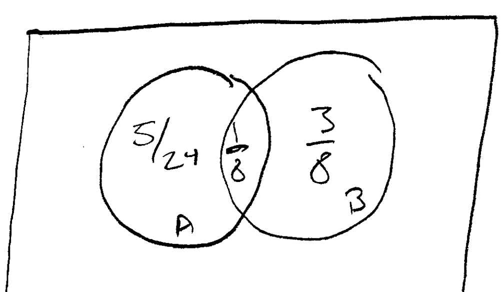
$P(A' \cap B) = 1/2$

(b) A is a subset of B



$P(A' \cap B) = 1/6$

(c) $P(A \cap B) = 1/8$



so $P(A' \cap B) = 3/8$

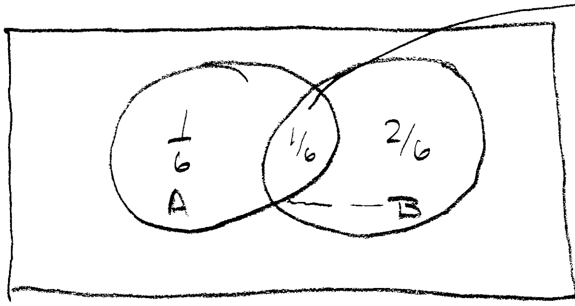
10) continued

d) A and B are indepedent

$P(A \cap B) = P(A) \cdot P(B)$

$P(A \cap B) = P(A)P(B)$

$=$



$P[A' \cap B] = \boxed{2/6}$