Program

Monday, August 23

Morning Session: Chaim Charach Memorial Session
Chairperson: Y. Zarmi

9:00–9:05	Opening Address
9:05–9:15	In Memory of Chaim Charach — Yair Zarmi, Paul Fife.
9:15–9:50	Fife, U. of Utah ***A Forced Motion by Curvature Problem Arising in Materials Science***
9:50–10:25	Novick-Cohen, Technion ***Relaxational Effects in Phase Separation***
10:25–11:00	Prigozhin, Ben-Gurion U. ***Nonlinear Dynamics of Aeolian Sand Ripples***
11:00–11:30	Coffee
11:30–12:30	Meron, Ben-Gurion U. ***Nonequilibrium Phase Separation***
12:30–13:05	Zaltzman, Ben Gurion U. ***Diffusional Mechanism of "Strong Selection" in Ostwald Ripening***
13:05–15:00	Lunch

Afternoon Session
Chairperson: A. Novick-Cohen

15:00–16:00 Coleman, Rutgers U.
Elastic Stability of Supercoiled Configurations of DNA Plasmids

16:00–16:30 Coffee

16:30–17:05 Zorski, Polish Academy of Science
Statistical Theory of Dislocations in 2-Dimensional Elastic Body

Tuesday, August 24

Morning Session
Chairperson: B. Coleman

9:00–10:00	Sniatycki, U. of Calgary ***Directional Control in Presence of Non-Holonomic Constraints***
10:00–10:35	Epstein, U. of Calgary ***On Some Geometric Material Structures***
10:35–11:05	Coffee
11:05–11:40	Drozdov, Institudte for Industrial Mathematics ***Large Physical Aging and Viscoelastic Behavior of Amorphous Glassy Polymers***
11:40–12:20	Muschik, Technische Universität, Berlin ***Mesoscopic Concept in Continuum Physics of Complex Materials***
12:20–13:00	Segev, Ben-Gurion U. ***Geometrical Aspects of Stress Theory***
13:00–15:00	Lunch

Afternoon Session
Chairperson: P. Fife

15:00–16:00 Druyanov, Hebrew U.
The Theory of Shakedown as a Base for Design

16:00–16:30 Coffee

16:30–17:20 Mainardi
Fractional Diffusion Processes and Related Random Walk Models

Evening: Optional tour to Tel-Aviv

Wednesday, August 25

Morning Session
Chairperson: H. Zorski

9:00–9:35	Trimarco, U. of Pisa ***Stresses and Energy-Stresses***
9:35–10:10	Craciun, Ovidius U. ***Prestressed Anisotropic and Isotropic Materials Containing a Crack***
10:10–10:45	Wozniak, Tech. U. Czstochowa ***Internal Variables in Dynamics of Periodic Systems***
10:45–11:15	Coffee
11:15–11:50	Elżanowski, Portland State U. ***On Non-Symmetric Deformations of Elastic Sphere***
11:50–12:40	Grinfeld, ETS Princeton ***The Stress Driven Instability in Elastic Crystals: Mathematical Models and Physical Manifestations***
13:00	Lunch

Afternoon: Walk in Jerusalem and conference dinner

Thursday, August 26

Morning Session
Chairperson: M. Epstein

9:00– 9:35	Kures, Tech. U. Brno ***Connections Attached to Lagrangians and the Hamiltonization***
9:35–10:10	Vashakmadze, Javakhishvili Tbilisi State U. ***On Two-dimensional Nonlinear Theory of Anisotropic Plates and Applications in Ocean Acoustic Problems***
10:10–10:45	Sellers, U. of East Anglia ***Theory of Solute Transport***
10:45–11:15	Coffee
11:15–11:50	de Botton ***On The Onset of Surface Instabilities in Orthotropic Composites***
11:50–13:00	Round Table Discussion
13:00–15:00	Lunch

Abstracts

Monday, August 23

Morning Session

A Forced Motion by Curvature Problem Arising in Materials Science

Paul C. Fife, *J. Cahn, C. Elliott & O. Penrose*

The authors' previous phase-field model for the phenomenon of diffusion-induced grain boundary motion is reduced by asymptotics to a free boundary problem involving motion by curvature with a forcing term dependent on the concentration of a solute substance which can also diffuse along the interface and be absorbed, during the motion, in the growing crystal. We thus have motion by curvature coupled to a diffusion process along the interface.

The problem for a steadily propagating solution is also formulated and existence proved, along with several properties of the solution.

Mathematics Department, University of Utah, 155 So. 1400 East, Salt Lake City, UT 84112-0090, USA.
Office: 801-581-6819 or 801-581-6851. Fax: 801-581-4148.
fife@math.utah.edu

Relaxational Effects in Phase Separation

Amy Novick-Cohen *& H. Rotstein*

We propose a new model which takes into account relaxational effects which can occur during phase separation. Such relaxational effects can arise as a result of a coupling to internal relaxational variables. Such effects might include i) structural relaxation in a viscous polymer melt undergoing phase separation or ii) slow relaxation of strain fields associated with lattice misfit of two consituents in a phase separating binary alloy. Our model represents a generalization of the approach of Jäckle, Binder, and Frisch in that we allow the chemical potential to have a time averaged response to all possible variations in the free energy. In an appropriate asymptotic limit, in the nonconserved case, this model yields sharp phase interface behavior in which the normal interfacial velocity is dictated by the time averaged mean curature.

Department of Mathematics, Dept. of Mathematics Technion-IIT Haifa 32000, ISRAEL.

amync@techunix.technion.ac.il

Nonlinear Dynamics of Aeolian Sand Ripples

Leonid Prigozhin

Aeolian sand ripples form familiar regular patterns on coastal beaches and desert floors and indicate instability of flat sand surface under the wind-induced transport and rearrangement of loosely packed sand grains. Despite the significant progress in understanding the nature of sand ripples major questions remain open; these involve the most interesting part of ripple formation, the nonlinear interactions that follow the initial instability. Previous research on sand ripples has generally relied on stochastic molecular dynamics simulation or upon simplified continuous models yielding unrealistic results at the nonlinear stage of ripple growth. By means of a new deterministic continuum mechanics model that seems to better describe the essential physics, we investigate the salient nonlinear properties of ripple formation. The model may be considered a modification of the proposed recently "two-variable" models for surface granular flow and accounts for the main types of sand transport: saltation, reptation, and creep of sand grains. The linear stability analysis of this model confirms that ripples grow because of the geometrical effect of greater impact and ejection flux on upwind-oriented slopes than on downwind-oriented slopes. Rolling of dislodged particles down the surface slopes and scattering of ejected reptating grains trajectories by an uneven sand surface have smoothing effect and tend to hinder the ripple growth. Taking all these effects into account seems crucial for modeling the nonlinear dynamics of sand ripples. Using this approach, we were able to simulate not only the development of a typical asymmetric ripple shape but also the evolution of sand ripple pattern characterized by the ripple wave-length growth. We suggest that the coarsening of ripple array occurs not via simple merger of ripples, as is usually assumed, but via soliton-like interactions with partial material exchange between interacting ripples.

Center for Environmental Physics, Blaustein Institute for Desert Research,
Midreshet Ben-Gurion, 84990 Israel.
leonid@bgumail.bgu.ac.il
www.cs.bgu.ac.il

Nonequilibrium Phase Separation

Ehud Meron

The strong nonlinear structures of systems driven far from equilibrium often give rise to multiplicity of stable states which can be regarded as "nonequilibrium phases". Spatio-temporal patterns involving domains of distinct nonequilibrium phases may behave very differently from analogous patterns in equilibrium phase transitions. This difference can be attributed to the absence of a variational principle, like the minimization of free energy, far from equilibrium. In particular, the nonvariational nature of nonequilibrium systems allows for an interface instability that gives rise to a state of spatio-temporal chaos involving spontaneous nucleation and annihilation of vortex pairs. In this presentation I will analyse this interface instability in two different contexts, reaction diffusion systems and periodically forced oscillatory systems, elucidate the mechanism by which it may lead to spatio-temporal chaos, and confront theoretical results with recent experiments carried out by Swinney and coworkers.

Blaustein Institute for Desert Research and the Physics Department, Ben Gurion University, Israel.

ehud@bgumail.bgu.ac.il

Diffusional Mechanism of “Strong Selection” in Ostwald Ripening

Boris Zaltzman

The purpose of this study was to show through a systematic asymptotic analysis that fluctuations, accounted for as a diffusional perturbation in the Lifshitz-Slyozov-Wagner (LSW) model of Ostwald Ripening, provides, as conjectured previously by Meerson, a ”strong” selection of the limiting solution, out of a one-parameter family of similarity solutions with a finite support, as the sole attractor of time evolution. Throughout the latter, the previously described weak selection of other similarity solutions of that family, by the initial conditions with finite supports, occurs as intermediate time asymptotics. The respective mechanism is traced first for a simple instance of the LSW model with linear characteristic equations (integer power in the particle growth rate law equals -1), beginning with the analysis of steady states in the perturbed problem in similarity variables and weak selection in the unperturbed problem, followed by a detailed asymptotic analysis of the time-dependent perturbed problem, and generalized next for an arbitrary integer power in the range [-1,2]. The approximate asymptotic solutions obtained are compared with the exact numerical ones.

Blaustein Institute for Desert Research, Ben Gurion University, Israel.
boris@bgumail.bgu.ac.il

Monday, August 23

Afternoon Session

Elastic Stability of Supercoiled Configurations of DNA Plasmids

Bernard D. Coleman

A plasmid is a DNA molecule in which the two strands of the Watson-Crick double helix (duplex) structure are intertwined closed curves for which the excess link ΔL (defined as the difference between the Gauss linking number of the strands in the present state and their total twist in a standard stress-free state) is a topological constant of importance in molecular biology. Recent research of David Swigon, Irwin Tobias, and the speaker has yielded a way of using exact solutions of Kirchhoff's equations of equilibrium for elastic rods of circular cross-section to calculate configurations of plasmids with the effects of impenetrability and self-contact forces taken into account. This research has yielded also methods of determining the regions of bifurcation diagrams (presented as graphs of ΔL versus the writhe W of the duplex axis) that correspond to configurations giving local minima to elastic energy. Primary bifurcation branches, i.e., branches that originate at circular configurations, are composed of configurations with D_m symmetry with $m = 2, 3, \ldots$. Among recent results are the following: (i) Emerging from the primary branch for which $m = 3$ are secondary branches whose configurations have C_2 symmetry, and which, in turn, can bifurcate to yield tertiary branches of configurations without symmetry. (ii) Whether or not self-contact occurs, a non-circular configuration in the primary branch with m = 2 is locally stable when for it the derivative $\partial L/\partial W$, computed along that branch, is strictly positive. (iii) For primary branches with $m > 2$, the condition $\partial L/\partial W > 0$ is not sufficient for stability; in fact contact-free configurations in those branches are always unstable. (iv) A rule has been found that relates the number of points of self-contact and the occurrence of intervals of self-contact to the magnitude of ΔL. (v) For reasons to be explained in the talk, the bifurcation diagram for a plasmid contains families of equilibrium configurations that are isolated from the circular configuration and hence are not in primary, secondary, or higher-order branches.

Rutgers University, Piscataway, New Jersey 08854-8058, USA.
bcoleman@stokes.rutgers.edu

Statistical Theory of Dislocations in 2-Dimensional Elastic Body

Henryk Zorski

Two-dimensional linear and isotropic elastic body contains a large number of straight dislocation lines (edge or screw) perpendicular to the body‘s plane, i.e., dislocations are point defects, each generating its own elastic field.

Derivation of the equations governing the motion of the body and the “dislocation fluid” (the latter is also a continuum) follows general procedures in the kinetic theory of gases or fluids. Thus, it is assumed that there is a dislocation density and an average velocity. The forces on and between dislocations (in our case simply between points in the plane) are the Peach-Koehler forces. The dislocation‘s inertia force has the Newtonian form with the field mass, in general anisotropic.

Since no kinetic equations for the dislocation fluid are considered in this paper, constitutive laws are introduced in their simplest forms, e.g. similarly to the pressure in a gas.

The final system consists of five partial differential equations for the elastic displacement (two unknowns), dislocation density (one unknown) and the dislocation velocity (two unknowns). Hyperbolicity conditions are derived. Simple solutions are examined,such as traveling waves. Displacements, strains and stresses are calculated.

In this procedure, generally, nonelastic constitutive laws are replaced by the couple: linear elastic constitutive laws and new degrees of freedom pertaining to the dislocation fluid.

Z.T.O.C. I.P.P.T., Ul. Scientokrzyoka 21, Warsaw 00-049, POLAND.
hzorski@ippt.gov.pl

Tuesday, August 24

Morning Session

Directional Control in Presence of Non-Holonomic Constraints

Jedrzej Sniatycki

Dynamics of a mechanical system with linear non-holonomic constraints is studied under the assumption that the symmetry group of the system acts properly and freely on the configuration space. The equations of motion have three components: reconstruction, gauge momentum and shape space equations. We study the possibility of controling solutions of the reconstruction equations by using control forces in the shape space.

Department of Mathematics and Statistics, University of Calgary, Calgary, Alberta, Canada, T2N 1N4.

sniat@math.ucalgary.ca

On Some Geometric Material Structures

Marcelo Epstein

The theory of material inhomogeneities is reformulated and extended to certain classes of non-uniform materials. It is shown that the relevant geometric structure is a groupoid whose structural group is the normalizer of the typical symmetry group. Conditions of integrability are then analyzed and compared with their counterparts for uniform materials. In certain important cases those conditions are identical, effectively extending the concept of homogeneity to non-uniform materials. It is expected that the results will have some relevance in the realm of Functionally Graded Materials.

Department of Mechanical Engineering, University of Calgary, Calgary, Alberta, Canada, T2N 1N4

epstein@fsa.enme.ucalgary.ca

Large Physical Aging and Viscoelastic Behavior of Amorphous Glassy Polymers

Aleksey D. Drozdov

The paper deals with constitutive equations for the (linear and nonlinear) viscoelastic response of amorphous polymers accounting for physical aging (structural relaxation) of disordered media subjected to thermal treatment. Annealing of polymeric glasses (slow changes in their mechanical and physical properties with time after temperature jumps) has attracted substantial attention in the past three decades, since it drastically affects residual stresses and dimensional stability of polymers and polymeric composites. Despite numerous studies of this phenomenon, no physical model can be mentioned which (i) is based on physically plausible hypotheses at the micro-level and (ii) adequately predicts observations in various (mechanical, dilatometric, calorimetric, dielectric, etc.) tests.

The work is concerned with novel constitutive equations based on the theory of cooperative relaxation in a version of the concept of traps. According to the Adam–Gibbs theory, relaxation of stresses in an amorphous polymer is modeled as sequential rearrangement of large clusters of strands (cooperatively rearranged domains, flow units or relaxing regions). A flow unit is thought of as a globule consisting of long chains, short chains and free volume clusters. A polymer is treated as an ensemble of mutually independent flow units which rearrange at random times being driven by thermal fluctuations. Referring to the transition-state theory, we treat a rearrangement event in the phase space as a hop of a flow unit trapped in its cage to some liquid-like state, where the relaxing region forgets its previous configuration. The energy landscape of a non-equilibrated glass is assumed to be fixed, whereas the position of the liquid-like state changes with time approaching the energy level corresponding to thermal equilibrium. Changes in the energy of the liquid-like state are determined by the fictive temperature which obeys an analog of the Tool equation with a characteristic time proportional to the average time of rearrangement. Stress–strain relations for viscoelastic response and governing equations for volume recovery are developed using the laws of thermodynamics. They are verified by comparison with observations in static and dynamic mechanical tests and in dilatometric experiments. Fair agreement is demonstrated between experimental data for polycarbonate, poly(arylene etherimide), poly(ether ether ketone) and poly(vinyl acetate) and results of numerical simulation.

Institute for Industrial Mathematics, 4 Hanachtom Street, Beer-Sheva, 84311 Israel.
aleksey@CS.bgu.ac.il

Mesoscopic Concept in Continuum Physics of Complex Materials

Wolfgang Muschik

The mesoscopic concept in continuum physics consists of extending the domain of the balance equations by a set of mesoscopic variables, so generating a fiber bundle, and of introducing a local distribution function of these variables as a statistical element. The balance equations defined on this fiber bundle are presented, order parameters describing the complex behaviour of the material as macroscopic fields are derived, and examples concerning liquid crystals and microcracks are discussed.

Institut für Theoretische Physik, Technische Universität Berlin, D-10623 BERLIN, Germany.

womu0433@itp0.physik.TU-Berlin.DE

Geometrical Aspects of Stress Theory

Reuven Segev *& Guy Rodnay*

A generalization of Cauchy's theory for the existence of stresses to the geometry of differentiable manifolds is presented using the language of differential forms. Body forces and surface forces are defined in terms of the power densities they produce when acting on generalized velocity fields. The velocity fields are sections of a vector bundle W over the m-dimensional material manifold S. Thus, a body force on a body R is an W^*-valued, m-form on R and a surface force is a W^*-valued, $(m-1)$-form on ∂R.

The balance law is written in terms of the total power expanded by forces and it is viewed as a boundedness or regularity assumption on the force functionals for the various bodies.

The normal to the boundary is replaced by the tangent space equipped with the outer orientation induced by outwards pointing vectors.

In the resulting Cauchy's theorem. stresses are modeled as $m-1$, covector valued forms. A stress induces a surface force by restriction to the tangent space to the boundary, while the outer orientation of the tangent space is taken into account. This operation, to which we refer as inclined restriction, uses a sign rule based on an orientation of the material manifold.

The special cases of volume manifolds and Riemannian manifolds are discussed. In the case of a volume manifold, it is shown how a tensor can represent a stress form using the volume element. Finally, the classical Cauchy formula is recovered for Riemannian manifolds.

Department of Mechanical Engineering, Ben-Gurion University, P.O.Box 653, Beer-Sheva 84105, Israel.

rsegev@bgumail.bgu.ac.il

Tuesday, August 24

Afternoon Session

The Theory of Shakedown as a Base for Design

Boris Druyanov *& Itzhak Roman*

The theory of shakedown (or adaptation) is capable of direct, e.g.without detailed analysis of deformation process, predicting of asymptotic mechanical behavior of elastic plastic bodies subjected thermo mechanical loading: Low Cycle Fatigue and Ratchetting. The theory was originated by Melan and Koiter in the framework of the elastic perfectly plastic model of material.During last decades the theory was well extended to include in its scope more realistic material properties: strain hardening, creep, and damage. These achievments made it possible direct predicting a wide spectrum of various phenomena occurring in solids as a result of cyclic thermo mechanical loading, and estimating the life time duration. Having the theory in hand, it is possible to employ it for design of machine parts and structures experienced cyclic thermo mechanical loading. This method provides us with an opportunity to avoid laborious computations which are sometimes impossible because of the lack of data related to the process of loading. The talk will review the key points of the shakedown theory. The typical formulations of the problems and discussion of main results will be given. The idea of design based on the theory will be presented and expounded in an example.

Graduate School of Applied Science, The Hebrew University of Jerusalem, Givat Ram Campus, 91904 Jerusalem, Israel.

borisd@vms.huji.ac.il

Fractional Diffusion Processes and Related Random Walk Models

R. Gorenflo[†] *&* ***F. Mainardi***[‡]

In recent years in physics and in other fields, e.g. stochastic economical processes, pseudo-differential evolutionary equations modelling processes of anomalous diffusion are becoming more and more popular.

Interesting mathematical models can be achieved by replacing in the common diffusion equation the second-order space derivative or the first-order time derivative by suitable pseudo-differential operators, which can be interpreted respectively as a fractional space derivative (in the sense of Riesz-Feller) and a fractional time derivative (in the sense of Caputo).

In the former case the resulting process is of Markov-type, mathematically a semigroup, which generates all Levy stable probability distributions. In the latter case the resulting process exhibits memory effects, which generates probability distributions with variance proportional to a power law of time.

Discrete models can be obtained by discretization in space and in time. By taking care in constructing these, they can be interpreted not only as difference schemes for approximating the solution of initial value problems, but also as random walk models for simulating particle paths by the Monte Carlo technique. A report is given on possible accesses to find consistent models, which generalize the classical Brownian motion related to the common diffusion equation.

Some references to our previous works:

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† Department of Mathematics & Computer Science, Free University of Berlin, Arnimallee 2-6, D-14195 Berlin, Germany.
gorenflo@math.fu-berlin.de.

‡ [Corresponding Author], Department of Physics, University of Bologna, Via Irnerio 46, I-40126 Bologna, Italy.
mainardi@bo.infn.it

Wednesday, August 25

Morning Session

Stresses and Energy-Stresses

Carmine Trimarco

The traction at the boundary of a continuum body leads almost straightforwardly to the Cauchy stress tensor. By contrast, forces acting on a 'defect' in a continuum rather address to the introduction of an energy-stress tensor. In elasticity, such a tensor, also known as the Eshelby stress tensor, plays a fundamental role in the theory of inhomogeneous and heterogeneous (i.e. non-uniform) materials. Surprisingly, the Cauchy stress tensor turns out to be eventually expressible in the form of an energy-stress tensor. Eshelby and Cauchy tensors play distinct physical roles and, in order to avoid misunderstandings, a discriminating criterion is desirable. In elasticity, one may note that the Cauchy stress is defined in the current framework. Thus, it is a full Eulerian tensor. By contrast, the Eshelby stress is a full Lagrangian one as it is completely defined in the referential framework. Unfortunately, such a criterion fails in certain circumstances. This is the case of liquid crystals whose mechanical behaviour is entirely described in the actual framework. As is known, the equilibrium of line-singularities in liquid crystals is governed by an energy-stress tensor: the so-called Ericksen tensor. The question may arise whether such a tensor is Cauchy-like or Eshelby-like; or, also, whether the two coincide in this and other specific cases. It will be shown that the distinct physical role of the two is preserved in any case, despite the fact that both exhibit the form of an energy stress tensor. Such distinctness survives also in form and can be [ascertained] revealed by a novel criterion, which appeals to dynamics.

The distinctness between the two can be ascertained by remarking. However, the two tensors play distinct physical roles. Basing on this remark a useful criterion can be established in order to avoid. As is known, the equilibrium (and possibly the motion) of line-singularities is ; Any other referential framework is retained redundant if not misleading

It will be shown that the distinctness for the two kinds of energy-stress holds true in any case and that it can be pointed out by a novel criterion, which appeals to dynamics.

Universitá di Pisa, Dipartimento Matematica Applicata “U. Dini”, Via Bonanno 25/B, I-56126 PISA, Italy.

trimarco@dma.unipi.it

Prestressed Anisotropic and Isotropic Materials Containing a Crack

Eduard-Marius Craciun† *& E. Soós*‡

We consider a prestressed orthotropic, linear elastic material. The material is unbounded and contains a right crack.

In the first part of our paper we generalized Sih's fracture criterion also for a prestressed, orthotropic or isotropic linear elastic material when the faces of the crack are acted by symmetrically distributed normal stress having constant value. We determine the direction of the crack propagation and we compare the obtained results with those existing in the framework of Griffith's and Irwin's classical theory.

In the second part we consider a nearly isotropic material without initial stresses. It contains a right crack acted by symmetrically distributed constant tangential stresses. We analyze the critical of the applied tangential for which the crack will start to propagate, according to Sih's generalized theory and to the classical Griffith-Irwin's theory.

In the third part we apply Sih's energetical fracture criterion to determine the critical incremental shear stress producing crack propagation in the third fracture mode, and the direction of crack propagation in a prestressed orthotropic elastic material and compare the obtained results with those given by the classical Griffith-Irwin brittle fracture theory.

In the last part of our paper we determine the direction of crack propagation and the critical values of the incremental stresses which produce the crack propagation in the mixed mode of classical fracture.

References

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† "Ovidius" University Constantza, Fac. of Mathematics Bd. Mamaia 124, 8700 Constantza, Romania.
mcraciun@ovidius.ct.ro

‡ Institute of Mathematics of the Romanian Academy, PO. BOX 1-764, Ro-70700, Bucharest, Romania.

Internal Variables in Dynamics of Periodic Systems

Czesaw Woniak *& Margaret Woniak*

It is known that the exact dynamic investigation of microperiodic materials and structures, based on solid mechanics equations in most cases is beyond reach even using computer methods. However, restricting analysis to the dominant signal wavelengths large compared with a microstructure length, various approximate approaches to dynamic problems have been proposed. The best known are those based on the homogenization of differential equations with periodic coefficients. Unfortunately, the homogenization methods are incapable of describing dispersion and attenuation phenomena observed in dynamics of composite materials and periodic structures. That is why the analysis of dynamic problems for periodic systems which retains these phenomena has been carried out in the framework of different alternative heuristic models such as the effective stiffness theories, the interacting continua models or the mixture theories. The aforementioned approaches and their applications were restricted mainly to investigations of certain special classes of material structures, e.g. laminates or fibre reinforced composites. In the last decade a unified treatment of the dynamic phenomena by the new averaging approach both to the discrete and continuum periodic material systems was developed. The proposed method introduces into the modelling procedure the concept of internal variables. So far, this concept was used mainly in formulation of constitutive relations for inelastic materials. In the analysis of microperiodic materials and structures the internal variables are extra unknown fields which together with the averaged displacement-type fields describe the dynamic behaviour of these systems. The characteristic feature of internal variables is that they are governed by ordinary differential equations involving only time derivatives of these variables. Hence the internal variables do not enter boundary conditions. Thus, the models under consideration are governed by the partial differential equations for averaged displacement-type fields coupled with the ordinary differential equations for internal variables. All these equations have constant coefficients the computation of which requires a solution to a certain periodic eigenvalue problem on the representative cell of the structure. The proposed models make it possible to investigate not only problems with wavelengths large compared with a microstructure size but also those with wavelengths of an order of the periods of microstructure.

The aim of lecture is twofold. First, we are to outline the general line of approach leading from the solid mechanics equations for microperiodic materials to the internal variable model relations. Second, we show some applications of the model to the analysis of special dynamic problems in both continuous and discrete periodic material structures. The physical reliability of some results will be also discussed.

Technological University of Czestochowa, Poland.
mwozniak@ck-sg.p.lodz.pl

On Non-Symmetric Deformations of Elastic Sphere

Marek Elżanowski

In this work we investigate a class of non-symmetric deformations of a neo-Hookean incompressible (nonlinear) elastic sphere. We consider a class of deformations of the form

$$r = f(R,\Theta), \quad \theta = g(R,\Theta), \quad \phi = \Phi,$$

were spherical coordinates in both deformed and undeformed state are employed. Using the approach and methodology of [1] we show that the results obtained there for a smaller than our class of non-symmetric deformations hold also for this broaden class. Namely, the three-dimensional equilibrium equations supplemented by the incompressibility constraint admit only the trivial solution, the radially symmetric deformations and a very particular class of non-symmetric, non-homogeneous deformations. This fact shows once again that for a neo-Hookean sphere broadening a class of admissible deformations does not produce any other than radially symmetric cavity as first reported in [2] for the radially symmetric deformations.

References

(1) D.A. Polignone-Warne and P.G. Warne, On non-symmetric deformations of an incompressible nonlinear elastic isotropic sphere, *J. Elasticity* 47 (1997), 85 – 100.
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Portland State University,Department of Mathematical Sciences, P.O.Box 751, Portland, OR 97207, U.S.A.
marek@mth.pdx.edu

The Stress Driven Instability in Elastic Crystals: Mathematical Models and Physical Manifestations

Michael A. Grinfeld

Equilibrium equations and stability conditions for the simple deformable elastic body are derived by means of considering minimum of the static energy principle. The energy is supposed to be sum of the volume (elastic) and the surface terms. The ability of changing relative positions of different material particles is taking into account and appropriate natural definitions of the first and second variations of the energy are introduced and calculated explicitly. Considering the case of negligible magnitude of the surface tension we establish that an equilibrium state of nonhydrostatically stressed simply elastic body (of any physically reasonable elastic energy potential and of any symmetry) possessing any small smooth part of free surface is always unstable with respect to relative transfer of the material particles along the surface. Surface tension suppresses mentioned instability with respect to sufficiently short disturbances of the boundary surface and thus probably can provide local smoothness of the equilibrium shape of crystal. We derive explicit formulas for critical wavelength for the simplest models of the internal and surface energies and for the simplest equilibrium configurations.We formulate also the simplest problem mathematical physics revealing peculiarities and difficulties of the problem equilibrium shape of elastic crystals and discuss possible manifestations of the above mentioned instability in the problems of crystal growth, materials science, fracture, physical chemistry and of the low temperature physics.

The Educational Testing Service, Mathematics Group, Bldg. N, Rosedale Rd, Princeton, 08541.
mgrinfeld@ets.org

Thursday, August 26

Morning Session

Connections attached to Lagrangians and the Hamiltonization

Miroslav Kureš

The contribution deals with the geometrical study of Lagrangian and Hamiltonian higher order formalisms. There is a simplicial structure on generalized velocities bundles and it facilitates the description of significant connections on them.

Department of Mathematics, Technical University of Brno, Technická 2, 616 69 Brno, Czech Republic.
kures@mat.fme.vutbr.cz

On Two-dimensional Nonlinear Theory of Anisotropic Plates and Applications in Ocean Acoustic Problems

Tamaz Vashakmadze[†*] *& Robert P. Gilbert*[‡*]

The problems of constructing two-dimensional models without using any assumptions on geometrical and physical characters due to the investigations of especially Ciarlet, Fridriechs, Koiter, Vekua, and Vorovich do not only have an independent meaning for the development of mathematical theory of anisotropic elastic plates but also create rigorous theory of elastic plates and shells in an applied sense for investigations on complete problems at continuous mechanics where the problems of solids are becoming part of them. With these connections, in this work we consider:

1.) Problems which enlarge and justify some results for anisotropic plates with one elastic symmetry plane and construct two-dimensional models of von Karman-Reissner types in term of so-called engineering coefficients: Young's moduli and Poisson ratios, without using asymptotic methods of Ciarlet-Fridriechs-Goldenweiser.

2.) Construct and justify two-dimensional nonlinear new models for some direct and inverse ocean acoustic problems with poro-thermo elastic inclusions.

† Javakhishvili Tbilisi State University, Chavchavadze Av. 1, Republic of Georgia.
tamaz@math.udel.edu
vasha@viam.hepi.edu.ge
‡ Dept. of Mathematical Sciences, University of Delaware Newark, DE 19716.
gilbert@ath.udel.edu
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Theory of Solute Transport

Eliot Fried[†] & ***Shaun Sellers***[‡]

A generalized continuum framework for the theory of solute transport in fluids is proposed and systematically developed. This framework rests on the introduction of a generic force balance for the solute, a balance distinct from the macroscopic momentum balance associated with the mixture. Special forms of such a force balance have been proposed and used going back at least as far as Nernst's 1888 theory of diffusion. Under certain circumstances, this force balance yields a Fickian constitutive relation for the diffusive solute flux, and, in conjunction with the solute mass balance, provides a generalized Smoluchowski equation for the mass fraction. Our format furnishes a systematic procedure for generalizing convection-diffusion models of solute transport, allowing for constitutive nonlinearities, external forces acting on the diffusing constituents, and coupling between convection and diffusion.

† TAM - 216 Talbot Lab. University of Illinois, 104 S. Wright Street Urbana, IL 61801, U.S.A.
‡ School of Mathematics, University of East Anglia, Norwich NR4 7TJ, England.
sellers@uea.ac.uk

On the Onset of Surface Instabilities in Orthotropic Composites

Gal deBotton

The work deals with the problem of surface buckling of reinforced solids by application of a simple model for composite materials, in which, the microscopic and macroscopic buckling mechanisms arise in a natural way. It is based on an analytical solution, that was obtained by deBotton and Schulgasser [J. Appl. Mech., 63], to the problem of surface instability of orthotropic solids subjected to uniaxial compression along the stiffest direction. A two phase model in which a single fiber of the reinforcement is bonded on top of an effective substrate and a three phase model involving a single fiber embedded within a layer of a matrix phase all bonded on top of an effective substrate are considered. In both cases, the predictions for the critical buckling loads are lower than those obtained by regarding the composite as a homogeneous material. It is also found that the buckling load and the critical wavelength are sensitive to the distance of the fibers from the surface of the composite, emphasizing a possible effect of small changes in the microstructure on the overall bulk properties of the composite.

Department of Mechanical Engineering, Ben-Gurion University of the Negev, Beer-Sheva 84105, Israel.

debotton@bgumail.bgu.ac.il

Authors Index