An Introduction

to

Applied Partial Differential Equations

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Contents

Preface

These notes are written for a one-quarter (pilot) course in elementary partial differential equations. It is assumed that the student has a good background in calculus, vector calculus, and ordinary differential equations. No prior knowledge of any partial differential equations concepts is assumed, nor any required. Some familiarity with the elementary theory of inner vector spaces would be an asset but is not expected. In fact, most of the needed concepts and facts are reviewed in the Appendix.

The main objective of this presentation is to introduce basic analytic techniques useful in solving most fundamental partial differential equations that arise in the physical and engineering sciences. The emphasis are placed on the formulation of a physical problem, deriving explicit analytic results, and on the discussion of properties of solutions. Although the proofs are usually omitted, the underlying mathematical concepts are explained and discussed at length.

The notes are divided into several short chapters and the Appendix. In Chapter 1 we discuss solutions to the equilibrium equations of one-dimensional continuous systems. These are formulated as boundary-value problems for scalar ordinary differential equations. The Green's function technique and the minimum principle are discussed. Chapter 2 deals with the diffusion equation, in particular, the heat propagation equation. In the last section of this chapter we briefly discuss the Burgers' equation. Solutions to a variety of homogeneous and inhomogeneous initial-boundary-value problems are derived using such analytic techniques as the separation of variables method and the concept of the fundamental solution. Laplace's equation and the wave equation are dealt with in Chapter 3 and 4, respectively. Once again, the separation of variables and the Fourier series methods are utilized. The Green's function technique is also researched. d'Alembert's solution of the wave equation is derived. An elementary discussion on the propagation of one-dimensional non-linear waves is presented. The Appendix consists of two parts. In the first part, the elements of the theory of inner product vector spaces are reviewed. The second part contains the presentation of the theory of Fourier series, and a short section on Fourier integrals.

Although the notes are as self contained as possible, students may find useful to consult some other texts like for example [**Bleecker and Csordas**], [**Boyce and DiPrima**], [**Keane**], [**Knobel**], and [**Davis**], among others.

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